Single Pure - Circles

Patrons are reminded that the tangent to a circle lies perpendicular to the radius *at that point*. Also remember that the angle subtended in a semi-circle is 90°.

1. Write down (or calculate) the equation of the circle with the desired properties.

   (a) The circle with centre $(0, -2)$ and radius 4. $\boxed{x^2 + (y + 2)^2 = 16}$

   (b) The circle with centre $(-5, 3)$ and radius 2. $\boxed{(x + 5)^2 + (y - 3)^2 = 4}$

   (c) The circle with centre $(2, -1)$ and radius $\sqrt{2}$. $\boxed{(x - 2)^2 + (y + 1)^2 = 2}$

   (d) The circle with centre $(1, -\frac{2}{3})$ and radius $3\sqrt{3}$. $\boxed{(x - 1)^2 + (y + \frac{2}{3})^2 = 27}$

   (e) The circle with centre $(5, 12)$ which passes through the origin. $\boxed{(x - 5)^2 + (y - 12)^2 = 169}$

   (f) The circle with centre $(0, 4)$ which passes through the point $(3, 0)$. $\boxed{x^2 + (y - 4)^2 = 25}$

   (g) The circle with centre $(1, -1)$ which passes through the point $(2, 1)$. $\boxed{(x - 1)^2 + (y + 1)^2 = 5}$

   (h) The circle with AB a diameter where $A = (1, 7)$ and $B = (4, 2)$. $\boxed{(x - \frac{5}{2})^2 + (y - \frac{9}{2})^2 = \frac{17}{2}}$

2. Find the centre and radius of the following circles.

   (a) $x^2 + y^2 = 49$. $\boxed{\text{Centre} = (0, 0), \text{Radius} = 7}$

   (b) $x^2 + y^2 = 20$. $\boxed{\text{Centre} = (0, 0), \text{Radius} = 2\sqrt{5}}$

   (c) $2x^2 + 2y^2 = 7$. $\boxed{\text{Centre} = (0, 0), \text{Radius} = \frac{\sqrt{14}}{2}}$

   (d) $(x - 1)^2 + y^2 = 50$. $\boxed{\text{Centre} = (1, 0), \text{Radius} = 5\sqrt{2}}$

   (e) $(x + 8)^2 + (y + 3)^2 = 1$. $\boxed{\text{Centre} = (-8, -3), \text{Radius} = 1}$

   (f) $(x - 3)^2 + (y + 7)^2 = 18$. $\boxed{\text{Centre} = (3, -7), \text{Radius} = 3\sqrt{2}}$

   (g) $(x - \frac{2}{3})^2 + (y + \pi)^2 + 1 = 29$. $\boxed{\text{Centre} = (\frac{2}{3}, -\pi), \text{Radius} = 2\sqrt{7}}$

   (h) $(-x - 2)^2 + y^2 = 45$. $\boxed{}$

   (i) $x^2 + y^2 - 2x = 0$. $\boxed{\text{Centre} = (1, 0), \text{Radius} = 1}$

   (j) $x^2 + y^2 - 10x + 14y - 7 = 0$. $\boxed{\text{Centre} = (5, -7), \text{Radius} = 9}$

   (k) $x^2 + y^2 + 6x - 8y + 1 = 0$. $\boxed{\text{Centre} = (-3, 4), \text{Radius} = 2\sqrt{6}}$

   (l) $x^2 + y^2 + x - 3y = 3$. $\boxed{\text{Centre} = (-\frac{1}{2}, \frac{3}{2}), \text{Radius} = \frac{\sqrt{22}}{2}}$

   (m) $2x^2 + 2y^2 = 2y + 8$. $\boxed{\text{Centre} = (0, \frac{1}{2}), \text{Radius} = \frac{\sqrt{17}}{2}}$

   (n) $x^2 + y^2 + \alpha x + \beta y = 0$. $\boxed{\text{Centre} = (-\frac{\alpha}{2}, -\frac{\beta}{2}), \text{Radius} = \frac{\sqrt{\alpha^2+\beta^2}}{2}}$

   (o) $px^2 + py^2 + qx + ry + s = 0$. $\boxed{\text{Centre} = (-\frac{q}{2p}, -\frac{r}{2p}), \text{Radius} = \frac{\sqrt{q^2+r^2-4p^2s}}{2p}}$

3. Find where the line (or curve) crosses the given circle.

   (a) $x + y = 3$ and $x^2 + y^2 = 5$. $\boxed{(1, 2) \text{ or } (2, 1)}$

   (b) $y = x - 1$ and $x^2 + (y - 3)^2 = 26$. $\boxed{(5, 4) \text{ or } (-1, -2)}$

   (c) $y = 2x - 1$ and $x^2 + y^2 = 1$. $\boxed{(0, -1) \text{ or } (\frac{4}{5}, \frac{3}{5})}$

   (d) $y = 3x - 1$ and $(x - 1)^2 + (y - 2)^2 = 10$. $\boxed{(0, -1) \text{ or } (2, 5)}$

   (e) $y = x + 1$ and $x^2 + y^2 = 9$. $\boxed{\left(\frac{-1+\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right) \text{ or } \left(\frac{-1-\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}\right)}$

(f) $y = x^2 - 2$ and $x^2 + y^2 = 4$. [Don't just dive in here! Think *before* writing.] $\boxed{(0, -2) \text{ or } (\sqrt{3}, 1) \text{ or } (-\sqrt{3}, 1)}$

4. Find where the circle $x^2 + (y - 1)^2 = 10$ crosses the x-axis. $\boxed{(3, 0) \text{ or } (-3, 0)}$

5. Find where the circle $(x + 1)^2 + (y + 2)^2 = 12$ crosses the y-axis. $\boxed{(0, -2 + \sqrt{11}) \text{ or } (0, -2 - \sqrt{11})}$

6. Find where the circle $(x - 2)^2 + (y - 5)^2 = 5$ intersects the circle $(x - 1)^2 + (y + 3)^2 = 50$. $\boxed{(0, 4) \text{ or } (\frac{48}{13}, \frac{46}{13})}$

7. Find the required tangents or normals.

   (a) Find the equation of the tangent to $x^2+y^2 = 25$ at the point $(3, 4)$ in the form $ax+by+c = 0$. $\boxed{3x + 4y - 25 = 0}$

   (b) Find the equation of the normal to $x^2+y^2 = 17$ at the point $(1, 4)$ in the form $ax+by+c = 0$. $\boxed{4x - y = 0}$

   (c) Find the equation of the tangent to $(x - 1)^2 + (y - 3)^2 = 2$ at the point $(2, 4)$ in the form $ax + by + c = 0$. $\boxed{x + y - 6 = 0}$

   (d) Find the equation of the tangent to $(x + 2)^2 + (y - 1)^2 = 5$ at the point $(0, 0)$ in the form $ax + by + c = 0$. $\boxed{2x - y = 0}$

   (e) Find the equation of the normal to $x^2 + y^2 - 6x + 2y = 15$ at the point $(-1, 2)$ in the form $ax + by + c = 0$. [Hint: Find the centre of the circle.] $\boxed{3x + 4y - 5 = 0}$

   (f) Find the equation of the tangent to $x^2 + y^2 + 4x + 2y = 15$ at the point $(2, 1)$ in the form $ax + by + c = 0$. $\boxed{2x + y - 5 = 0}$

   (g) Find the equation of the tangent to $x^2 + y^2 + 4x - 4y = 5$ at the point $(0, 5)$ in the form $ax + by + c = 0$. $\boxed{2x + 3y - 15 = 0}$

   (h) Find the equation of the normal to $x^2 + y^2 - x + 3y = 4$ at the point $(1, 1)$ in the form $ax + by + c = 0$. $\boxed{5x - y - 4 = 0}$

   (i) Find the equation of the tangent to $x^2 + y^2 - 5y = 25$ at the point $(-1, -3)$ in the form $ax + by + c = 0$. $\boxed{2x + 11y + 35 = 0}$

   (j) Find the equation of the normal to $x^2 + y^2 - x + 2y - 26 = 0$ at the point $(2, 4)$ in the form $ax + by + c = 0$. $\boxed{10x - 3y - 8 = 0}$

8. Find the equations of the tangents drawn from the point $(4, -3)$ to the circle $x^2 + y^2 = 5$. Give your answers in the form $ax + by + c = 0$. $\boxed{2x + y - 5 = 0, 2x + 11y + 25 = 0}$

9. Find the value of k such that $(x - 3)^2 + (y + k)^2 = 16$ merely touches the x-axis. $\boxed{k = \pm 4}$

10. Find the value(s) of k such that $y = k - x$ lies tangent to the circle $x^2 + y^2 - 2x + 2y = 1$. $\boxed{k = \pm\sqrt{6}}$

11. Find the value(s) of k such that $y = 2x + k$ lies tangent to the circle $x^2 + y^2 - 2x + y = 1$. $\boxed{}$

12. You are given that $A = (0, 0)$, $B = (2, 2)$ and $C = (-4, 4)$.

    (a) Calculate the gradients of AB, AC and BC. $\boxed{1, -1, -\frac{1}{3}}$

    (b) By considering said gradients, state (with reason) which of AB, AC and BC are perpendicular. $\boxed{AB \text{ and } AC}$

    (c) Hence state (with reason) which of AB, AC or BC is the diameter of the circle passing though A, B and C. $\boxed{BC}$

    (d) Find the equation of the circle passing through A, B and C. $\boxed{(x + 1)^2 + (y - 3)^2 = 10}$

13. Using precisely the same method as the previous question, find the equation of the circle through the following sets of points.

    (a) $R = (1, 0)$, $S = (3, 0)$, $T = (2, 1)$. $\boxed{(x-2)^2 + y^2 = 1}$

    (b) $P = (2, 0)$, $Q = (1, -1)$, $R = (3, -3)$. $\boxed{(x-\frac{5}{2})^2 + (y+\frac{3}{2})^2 = \frac{5}{2}}$

    (c) $K = (2, -3)$, $L = (-2, -1)$, $M = (-1, 1)$. $\boxed{(x-\frac{1}{2})^2 + (y+1)^2 = \frac{25}{4}}$

    (d) $A = (8, -1)$, $B = (3, 4)$, $C = (2, 1)$. $\boxed{(x-\frac{11}{2})^2 + (y-\frac{3}{2})^2 = \frac{25}{2}}$

14. Now you've got to find a new method! Find the circle that passes through the following.

    (a) $A = (0, 0)$, $B = (2, 0)$, $C = (4, -2)$. $\boxed{(x-1)^2 + (y+3)^2 = 10}$

    (b) $A = (2, 2)$, $B = (4, 3)$, $C = (6, 9)$. $\boxed{(x-\frac{1}{2})^2 + (y-\frac{15}{2})^2 = \frac{65}{2}}$

    (c) $U = (-1, 1)$, $V = (2, -1)$, $W = (-2, 0)$. $\boxed{(x+\frac{1}{10})^2 + (y+\frac{9}{10})^2 = \frac{221}{50}}$

    (d) $P = (0, 0)$, $Q = (a, 0)$, $R = (1, 1)$. $\boxed{(x-\frac{a}{2})^2 + (y+\frac{a-2}{2})^2 = \frac{a^2-2a+2}{2}}$